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Branes ending on branes in a tachyon model

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Abstract: In a tachyon model proposed by Minahan and Zwiebach and derived in the boundary string field theory, we construct various new solutions which correspond to nontrivial brane configurations in string theory. Our solutions include Dp-D$(p-2)$ bound states, (F, Dp) bound states, string junctions, D$(p-2)$-branes ending on a Dp-brane, D$(p-2)$-branes suspended between parallel Dp-branes and their non-commutative generalizations. We find the Bogomol'nyi bounds and the BPS equations for some of our solutions, and check the physical consistency of our solutions with the D-brane picture by looking at the distributions of their energies and RR-charges in space. We also give conjectures for a few other brane configurations.

Keywords: Superstrings and Heterotic Strings, D-branes, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Following the Sen's conjectures [1]–[3], the tachyon condensation in string theory has been attracting considerable attention for recent a few years among many other developments in string/M theory. In particular the tachyon condensation in open string (field) theory provides us with an intriguing scenario [3]: all the string/brane physics could be reproduced via the tachyon condensation, starting with an unstable 9-brane system, either non-BPS D9-branes in type IIA or D9-antiD9 pairs in type IIB. Significant efforts along this line have been made in several approaches such as the cubic string field theory [4], the boundary string field theories (BSFT) [5]–[15] and the non-commutative tachyons [16]–[20], giving good amount of positive evidences for this fascinating scenario.

Among others, Minahan and Zwiebach proposed the field theory models of the tachyon [21]–[24]. For a tachyon model of an unstable D9-brane in type IIA, they provided the action [24],

$$S = -\mathcal{T}\int d^{10}x\; e^{-T^2/a}\left(1 + (\partial_\mu T)^2 + \frac{1}{2}F_{\mu\nu}^2\right). \tag{1.1}$$

This model, even with the inclusion of appropriate fermion couplings, enjoys desirable properties as an effective theory of a non-BPS D9-brane, such as the appearance of the lower dimensional D-branes as kinks or lumps with the linear profile of the tachyon field and of discrete spectra with an equal spacing for the fluctuations about the soliton solutions, which are all in favor of stringy behavior of D-branes. This result turned out not to be an accident, for the action (1.1) can actually be thought of as the two-derivative truncation of the action obtained from the BSFT for unstable D-branes [25, 26].[1]

Up to now only the flat D-branes are constructed as kinks or lumps in the tachyon models. So there remains the bulk of enterprises to explore the zoology of brane configurations as solitons in the tachyon models, such as the bound states of branes, i.e. branes within branes, intersecting branes and branes ending on branes, etc., which is necessary to put forward the conjecture that all the branes can be made as kinks or lumps out of unstable $D9$-branes.

In this paper we will make use of the action (1.1) and study the solutions, such as kinks, lumps and their generalizations, which correspond to various brane configurations in superstring theory. We found a variety of nontrivial BPS brane configurations, including (i) a D8-D6 bound state, (ii) a (F, D8) bound state, (iii) a string junction, (iv) D6-branes ending on a D8-brane, (v) a D6-brane suspended between two parallel D8-branes and (vi) their non-commutative generalizations.

The solutions obtained in this paper are expected to persist in the inclusion of the higher derivative corrections computed in the BSFT, as our solutions are BPS. The tensions and the RR-charges of our solutions will be reproduced in exact agreement only when working on the full BSFT action. Indeed the analysis of the full-fledged BSFT action gives the exact results and will be presented in our forthcoming paper [27].

2. Brane bound states and string junctions

We shall start by constructing a few simple solutions of the action (1.1)[2] representing the brane bound states and string junctions [28, 29]. Before discussing these

[1]Precisely speaking, there could be an additional term, proportional to $T^2(\partial T)^2$, whose coefficient depends on the renormalization scheme and a certain choice of which can give the correct tachyon mass at the level of the tree level action [14].

[2]The BSFT action in the two-derivative truncation (again up to a term remarked in the previous footnote) is given in [25, 26, 11, 12, 14],

$$S = -T_{\rm D9}\int d^{10}x\, e^{-2\pi\alpha'\widetilde{T}^2}\left(1+4\pi\alpha'^2\log 2(\partial_\mu\widetilde{T})^2+\pi^2\alpha'^2\widetilde{F}_{\mu\nu}^2\right). \tag{2.1}$$

If we rescale the tachyon field and the field strength in an appropriate manner, the following identification

$$a = 4\alpha'\log 2, \quad \mathcal{T} = T_{\rm D9} \tag{2.2}$$

solutions, let us recall the basic property of a known solution which represents a flat D8-brane [11, 23]. The equations of motion for this system (1.1) are

$$\partial_\mu^2 T + \frac{1}{a}T\left(1 - (\partial_\mu T)^2 + \frac{1}{2}F_{\mu\nu}^2\right) = 0\,, \tag{2.3}$$

$$\partial_\mu(e^{(-T^2/a)}F_{\mu\nu}) = 0\,. \tag{2.4}$$

The solution for the flat D8-brane is given by

$$T = qx_9, \quad A_\mu = 0\,, \tag{2.5}$$

where a constant q is fixed to be 1. Due to the form of the tachyon potential in the action (1.1), the kink solution is actually described by a linear tachyon profile, since the potential minima are located at $T = \pm\infty$. The constant q would be infinite if we included the higher derivative corrections computed in the BSFT. In the derivation of the BSFT action, the tachyon profile is assumed to be a linear function, so the solution (2.5) is consistent with this fact. In this paper, we will often assume the intuition based on the fact that the constant q is actually going to infinity when the higher derivative corrections are included as in the BSFT.

Now the energy of the kink solution (2.5) is given by

$$\mathcal{E} = \mathcal{T}\int d^9x\, e^{-T^2/a}\left(1 + (\partial_i T)^2 + \frac{1}{2}F_{ij}^2\right) = \mathcal{T}\int d^9x\, 2e^{-T^2/a} = 2\sqrt{a\pi}V_{\rm D8}\mathcal{T}\,, \tag{2.6}$$

where $V_{\rm D8}$ is the volume of a D8-brane. Therefore the energy density is localized at $x_9 = 0$ which represents the position of the kink. If the constant q gets larger and larger in the inclusion of the higher derivative corrections, the localization of the kink becomes sharper and sharper, and after all the gaussian distribution supplied by the tachyon potential is replaced by a delta-function .

We can compute the Ramond-Ramond (RR) charge of the kink solution (2.5) by adopting the formula for the Chern-Simons (CS) term given[3] in [25, 26]:

$$S_{\rm CS} = \mathcal{T}{\rm Tr}\int e^{-T^2/a}C \wedge \exp[a_1 F + a_2 DT]\,, \tag{2.7}$$

where a_1 and a_2 are constant numbers. Substituting the solution (2.5), one obtains the D8-brane charge

$$S_{\rm CS} = a_2\mathcal{T}\int C^{(9)} \wedge dT e^{-T^2/a} = a_2\sqrt{a\pi}\mathcal{T}\int_{V_{\rm D8}} C^{(9)}\,, \tag{2.8}$$

which is also localized on the position of the kink, as it should be.

gives the action (1.1). When comparing our results obtained in this paper with those in the BSFT, these tuned parameters (2.2) are to be adopted. However the two derivative truncation employed in this paper will not provide the correct values of energies and RR-charges expected from the well-known superstring results. We will see the exact agreement with the expected results, when taking into account the higher derivative corrections in our forthcoming paper [27].

[3] Again we have rescaled the tachyon field and the gauge fields. Thus in comparison with the BSFT result, we have to choose $a_1 = \sqrt{2}$ and $a_2 = \sqrt{\pi/2\log 2}$.

2.1 D8-D6 bound state

If we allow a constant field strength on the parent unstable D9-brane, the constant field strength will be inherited onto the kink as well. This corresponds to the D8-brane containing lower dimensional branes on its worldvolume. These lower dimensional branes are smeared out so that the distribution of the charge is uniform on the D8-brane worldvolume.

Let us consider this solution more concretely. When we turn on the constant field strength F_{78}, the equations of motion are satisfied by a linear profile for the tachyon T as

$$T = \sqrt{1 + F_{78}^2} x_9 \,. \tag{2.9}$$

This solution is regarded as a D8-D6 bound state. Actually the RR-charge is evaluated as

$$\begin{aligned} S_{\rm CS} = a_1 a_2 \mathcal{T} \int d^7x \, e^{-T^2/a} C^{(7)}_{0123456} \wedge (F_{78} dx_7 \wedge dx_8) \wedge (\partial_9 T) dx_9 + \\ + a_2 \mathcal{T} \int d^9x \, e^{-T^2/a} C^{(9)}_{012345678} \wedge (\partial_9 T) dx_9 \,. \end{aligned} \tag{2.10}$$

Therefore the constant field strength indicates that the D6-brane charge uniformly distributed over the 78 plane, as can be seen from the first term. The second term is the D8-brane charge which coincides with that of the previous example.

The energy of this bound state is evaluated as

$$\mathcal{E}_{\rm D8D6} = \sqrt{1 + F_{78}^2} \, \mathcal{E}_{\rm D8} \,. \tag{2.11}$$

This energy (2.11) gives the correct ratio between the bound state energy[4] of a D8- and D6-branes and that of a D8-brane [30].

When we rotate the solution (2.9) in the 69 plane with the angle $\tan\theta = F_{78}$, we can obtain another expression for the D8-D6 bound state as

$$F_{78} = \text{const.} \,, \qquad T = x_9 + F_{78} x_6 \,. \tag{2.12}$$

This rotated form turns out to be useful for later use, when we study the non-commutative monopole in section 4.3.

2.2 (F, D8) bound state and string junction

When we turn on an electric field instead of a magnetic field, we have an (F, D8) bound state [31, 32] solution:

$$F_{06} = \text{const.} \,, \qquad T = \sqrt{1 - F_{06}^2} \, x_9 \,. \tag{2.13}$$

[4]Even though we are using the two-derivative truncation of the full BSFT action, the result (2.11) seems to incorporate the Born-Infeld result of [30]. This might be due to the fact that the configuration we considered is supersymmetric.

The energy of this kink solution is calculated as

$$\mathcal{E}_{(\mathrm{F,D8})} = \frac{1}{\sqrt{1-F_{06}^2}} \mathcal{E}_{\mathrm{D8}} \,, \tag{2.14}$$

where the front factor originates from the tachyon potential $e^{-T^2/a}$ in the action after the integration over x_9. The energy (2.14) is in agreement with the ordinary Born-Infeld analysis of the energy of this bound state: the Hamiltonian density of the Born-Infeld system of a BPS D8-brane with a constant electric field is given by

$$\mathcal{H} = \Pi^\mu \dot{A}_\mu - \mathcal{L} = \frac{1}{\sqrt{1-F_{06}^2}} \mathcal{H}_{D8} \,, \tag{2.15}$$

where Π^μ is the conjugate momentum for the gauge fields, defined by $\Pi^\mu = \delta\mathcal{L}/\delta\dot{A}_\mu$ as usual.

Now we will proceed to the construction of a string junction [28, 29] in the tachyon model. Let us start with a string junction on a 2-dimensional plane spanned by x_6 and x_9, and take T-dualities along 1234578 directions. Then we arrive at the junction composed of a D8-brane, a (F, D8) bound state and the fundamental strings, instead of a D1-brane, a (F, D1) bound state and the fundamental strings. Now we can utilize the D8-brane solution (2.13) as a component of the junction. Hereafter we will call this T-dualized string junction simply as junction.

To construct this junction solution in the tachyon model, it is useful to refer to the construction given in [28]. Assuming that a constant electric field F_{06} is turned on only in the region $x_6 \geq 0$, we have the solution (2.13) of the (F, D8) bound state in this region. Now in the other region $x_6 < 0$, there is no electric field turned on, so we have only a pure D8-brane instead of a (F, D8) bound state. If we connect these two solutions at $x_6 = 0$, we will have a junction formed at this point, as is similar to [28]. Due to the force balance at the junction point, the pure D8-brane in the region $x_6 < 0$ must be tilted relative to the (F, D8) bound state. Thus, in this region, we put the ansatz

$$T = q_1 x_9 + q_2 x_6 \qquad (x_6 < 0) \,. \tag{2.16}$$

Substituting this into the equations of motion, we obtain the constraint

$$q_1^2 + q_2^2 = 1 \,. \tag{2.17}$$

Since two kink solutions (2.13) and (2.16) should be connected on a plane $x_6 = 0$, we obtain $q_1 = \sqrt{1-F_{06}^2}$ and $q_2 = F_{06}$. (Another possibility $q_2 = -F_{06}$ represents the opposite orientation of the junction.)

These two kinks are localized around the curves determined by the equation $T = 0$: one is on $x_9 = 0$ ($x_6 \geq 0$), and the other is on $\sqrt{1-F_{06}^2}x_9 + F_{06}x_6 = 0$ ($x_6 < 0$). It is easy to see that this is the same as the configuration given in [28], if we rotate the whole configuration on 69 plane by θ where $\sin\theta = F_{06}$. As claimed in [28], we have to assume the existence of the invisible fundamental strings stuck to the junction point.

Note that to obtain the uniform electric field only in the half of the 69 plane, we have to align positive electric charges on a plane $x_6 = 0$ and negative charges on $x_6 = +\infty$ (assuming $F_{06} > 0$). The electric flux covers the entire half of the plane, $x_6 \geq 0$. However the "effective" electric flux is confined only on the surface $\sqrt{1-F_{06}^2}x_9 + F_{06}x_6 = 0$. This is because the exponential factor of the tachyon potential in front of the gauge kinetic term in the action (1.1) is very small outside this surface. Thus the fundamental strings can be found only on the (F, D8) bound state. We will make a comment on unseen fundamental strings in the discussion.

It is straightforward to generalize the above junction configuration to the case with multiple D8-branes [33]. It is simply done by considering multiple D9-branes from the beginning. Let us consider the junction, (0,2)-(p,-1)-($-p$,-1), as an example. This can be obtained from two non-BPS D9-branes, and the solution is easily found as

$$T = \begin{cases} \sqrt{1-p^2}\, x_9 \mathbb{1} & (x_6 \geq 0) \\ \sqrt{1-p^2}\, x_9 \mathbb{1} + p\sigma_3 & (x_6 < 0) \end{cases}, \quad F_{06} = \begin{cases} p\sigma_3 & (x_6 \geq 0) \\ 0 & (x_6 < 0) \end{cases}. \tag{2.18}$$

To realize this configuration, we aligned electric charges on the planes $x_6 = 0$ and $x_6 = +\infty$ appropriately for each of the σ_3 component of the gauge fields.

3. Branes ending on branes

The branes ending on branes are one of the most interesting brane configurations. For instance, it is known that D6-branes can end on a D8-brane [34]. It is difficult to construct such configurations in the supergravity, while neat solutions are available in the effective gauge theories of BPS D-branes and they have been known as BIons [35, 36]. In this section we will look for a solution akin to BIons in the tachyon model (1.1).

3.1 A spike (kinky-lump) solution

An important feature of the kink solution (2.5) is that when a constant q becomes infinitely large in the inclusion of the higher derivative terms, the equation $T = 0$ exactly gives the location of D-branes sharply localized on this surface. Thus it is quite plausible to make the following ansatz for the solution representing the branes ending on branes:

$$T = q\left(x_9 - \frac{s}{r}\right), \tag{3.1}$$

where q and s are constant parameters, and $r = \sqrt{x_6^2 + x_7^2 + x_8^2}$. This is an exact analogy of the BIon which is given by $\Phi = s/r$ with Φ being a scalar field on the D8-brane that represents the displacement of the D8-brane, as the equation $T = 0$

with the above ansatz (3.1) describes the same bending $x_9 = s/r$ of the D8-brane as that for the BIon. Thus the above tachyon profile (3.1) gives a spike solution. The parameter s will remain a non-zero constant even in the inclusion of the higher derivative corrections, for it is observed that a constant s is not vanishing in the analysis of the Born-Infeld action and its higher derivative inclusions as in [35, 37].

Let us substitute the above ansatz (3.1) into the equations of motion (2.3) and (2.4). Noticing that $(\partial_\mu T)^2 = q^2(1+s^2/r^4)$, one can easily see that the constant 1 in the parenthesis of the tachyon equations of motion (2.3) is canceled if $q=1$. This is precisely analogous to the kink solution of a D8-brane (2.5). There is, however, another contribution $(\partial_i T)^2$ $(i=6,7,8)$, coming from a nontrivial deformation of the tachyon profile. This contribution is actually cancelled by turning on the gauge field strength as

$$B_i = qs\frac{x_i}{r^3} = \partial_i\left(\frac{-qs}{r}\right). \tag{3.2}$$

Here B_i is a magnetic field. This form of the magnetic field is the same as that of the BIon. The gauge equations of motion (2.4) are also satisfied, as the linear dependence on x_9 in the tachyon field is irrelevant to these equations and the contribution from the magneto-static potential $1/r$ is easily found to be cancelled by those from the magnetic fields on simple symmetry grounds.

Hence we have obtained a nontrivial solution in the tachyon model (1.1), which represents a D6-brane (0123459) ending on a D8-brane (012345678), in an exact analogy with the BIon.

The tachyon solution (3.1) is very similar to the one obtained in a supersymmetric sigma model with a hyper-Kähler target space [38]. The authors of [38] constructed "kinky-lump" solution which describes branes ending on a brane. In their model the tachyon minima are placed in the finite distance instead of the infinite distance as in our case. Hence the kink solution is described not by a linear function as qx_9 but by a hyperbolic tangent function. Also in [38], the branes are ending on a two dimensional space, instead of a three dimensional space in our case. Thus they have $\log|r|$ as a deformation of the tachyon profile, instead of $1/r$ in our solution.

3.2 The distribution of RR-charge

To check the physical consistency of our solution, we will look at the RR-charge. Naively one might think that the distribution of the magnetic field (3.2) is somewhat strange, for it has no dependence on the x_9 direction in which we expect the D6-branes are extended. If the D6-branes are ending on a D8-brane, the D6-brane charge should be distributed on a semi-infinite plane $x_9 > 0$ and $x_{678} = 0$.

This can be explicitly checked by using the formula (2.7) for the CS term. Let us substitute our spike solution (3.1) and (3.2) into the CS term

$$S_{\rm CS} = \mathcal{T}\int e^{-T^2/a} C^{(6)} \wedge \left(a_1 a_2 dT\wedge F + a_2^3 dT\wedge dT\wedge dT\right). \tag{3.3}$$

The second term $(dT)^3$ is vanishing, while the first term $dT \wedge F$ takes the form[5]

$$dT \wedge F = q\frac{s^2}{r^4} dx_6 \wedge dx_7 \wedge dx_8 \,. \tag{3.4}$$

Now the D6-brane charge can be read off from

$$\begin{aligned} a_1 a_2 \mathcal{T} \int d^6x \int dx_9 \; C^{(6)}_{0123459} \int dx_6 dx_7 dx_8 \; e^{-T^2/a} \; q \, \frac{s^2}{r^4} &= \\ = 4\pi^{3/2} a_1 a_2 \mathcal{T} \int d^6x \int dx_9 \; C^{(6)}_{0123459} \; S(qx_9/\sqrt{a}) \,, \end{aligned} \tag{3.5}$$

where we defined a smeared step function

$$S(x) \equiv \frac{1}{2}\left(1 + \mathrm{erf}(x)\right) , \tag{3.6}$$

and $\mathrm{erf}(x)$ is the error function defined as

$$\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x du \; e^{-u^2} . \tag{3.7}$$

Thus the CS coupling (3.5) implies that the D6-brane charge is distributed only on the semi-infinite plane $x_9 > 0$ ($x_{678} = 0$), as we expected (here we have assumed $s > 0$ for simplicity). The distribution of the D6-brane charge is sharpened when q gets larger and in the $q \to \infty$ limit the smeared step function $S(qx_9/\sqrt{a})$ approaches to the step function $\theta(x_9)$.

3.3 The Bogomol'nyi bound

Since the form of the action (1.1) itself is similar to the one of the ordinary Scalar-Maxwell theory except for the overall exponential factor, it is easy to guess the existence of the Bogomol'nyi bound in the tachyon model. Actually when we turn only on the tachyon and the magnetic fields in the x_{678} directions, the energy is nicely arranged into

$$\begin{aligned} \mathcal{E} = \mathcal{T} V_{12345} \int d^4x \; e^{(-T^2/a)} \left[(1 - \partial_9 T)^2 + (\partial_i T - B_i)^2 \right] + \\ + \mathcal{T} V_{12345} \int d^4x \; 2e^{(-T^2/a)} \left[\partial_9 T + \partial_i (T B_i) \right] . \end{aligned} \tag{3.8}$$

Thus if the configuration satisfies the BPS equations

$$\partial_9 T = 1 \,, \qquad \partial_i T = B_i \qquad (i = 6, 7, 8) \,, \tag{3.9}$$

[5] Here we have omitted the term proportional to dx_9 which will be vanishing after the integration over $dx_6 dx_7 dx_8$, assuming that the RR 7-form is independent of x_6,x_7 and x_8.

the energy is bounded by the topological quantities:

$$\begin{aligned}\mathcal{E} &= 2\sqrt{\pi a}\mathcal{T}V_{12345}\int d^3x\,\left[S\left(\frac{T}{\sqrt{a}}\right)\right]_{x_9=-\infty}^{\infty} + \\ &\quad +2\sqrt{\pi a}\mathcal{T}V_{12345}\int dx_9\int_{r=\infty} dS_i\left(B_i S\left(\frac{qx_9}{\sqrt{a}}\right)\right) \\ &= 2\sqrt{\pi a}\mathcal{T}V_{\mathrm{D8}} + 8\pi s\sqrt{\pi a}\mathcal{T}V_{12345}\int dx_9\ S(qx_9/\sqrt{a}). \end{aligned} \tag{3.10}$$

The first term gives the standard kink charge which corresponds to the energy of the D8-brane[6] extended along x_{678} in accord with (2.6). The second term is more interesting. This provides the energy of the D6-brane extended along the x_9 axis and terminated at $x_9 = 0$, as can be seen from the smeared step function S given above.

One may notice that the BPS bound (3.8) gives precisely the same expression as that of the RR-charges (3.3). This is similar to the case of the BPS D-branes in that the BPS bounds obtained from the Born-Infeld action is equal to the CS term for the RR-fields. This property is characteristic of the supersymmetric D-branes, implying the plausibility of the BPS bound we have found above.

Before closing this section, we would like to make a comment on multiple BIons. As long as the BPS equations (3.9) are satisfied, we can obtain the result similar to that we have found in this section. Thus, for example, it is possible to get a solution representing N D6-branes stuck to a D8-brane:

$$T = x_9 - \sum_{i=1}^{N}\frac{s^{(i)}}{|\mathbf{x}-\bar{\mathbf{x}}^{(i)}|}\,, \qquad B_i = \partial_i T\,. \tag{3.11}$$

The parameters $s^{(i)}$ can be either positive or negative, depending on which the i-th D6-brane is elongated either to $x_9 = +\infty$ or $-\infty$. The parameters $\bar{\mathbf{x}}^{(i)} = (\bar{x}_6^{(i)}, \bar{x}_7^{(i)}, \bar{x}_8^{(i)})$ indicates the location where the i-th D6-brane is stuck to the D8-brane. The evaluation of the energy and the RR-charges gives the expected result.

4. A brane suspended between two parallel branes

As a generalization of the solution of the previous section, we will discuss a solution of a D6-brane suspended between two parallel D8-branes. This brane configuration is constructed as the 'tHooft-Polyakov monopole in SU(2) Yang-Mills-Higgs theory [39, 40].

4.1 An explicit solution

To obtain two parallel D8-branes, we have to prepare at least two non-BPS D9-branes. In this section we will consider only two D9-branes. Each BPS D8-brane

[6] The first term becomes subtle at the singular point $r = 0$. We cannot define the energy there, since the D8-brane surface is placed at the infinity.

is given by a kink on each D9-brane via the tachyon condensation with the linear tachyon profile. We employ the following non-Abelian action with the adjoint tachyon and the gauge fields

$$S = -\mathcal{T}\ \mathrm{Str}\int d^{10}x e^{-T^2/a}\left(\mathbb{1} + (D_\mu T)(D^\mu T) + \frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right), \tag{4.1}$$

where Str denotes the symmetrized trace in that we symmetrize the U(2) matrices with respect to T^2 (in $e^{-T^2/a}$), $D_\mu T$ and $F_{\mu\nu}$. We adopt the standard normalization for the generators of the gauge group U(2), whereas the 2×2 unit matrix $\mathbb{1}$ is not normalized in that way. This action (4.1) can be thought of as the two derivative truncation of the action of multiple non-BPS D9-branes computed in the BSFT [25, 26] up to the ordering ambiguity.

The equations of motion derived from the above action are

$$\frac{\delta S}{\delta T} = \sum_\sigma\left[\left(2D_\mu D^\mu T + \frac{2}{a}T\left(\mathbb{1} - D_\mu T D^\mu T + \frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right)\right)e^{-T^2/a}\right] = 0\,, \tag{4.2}$$

$$\frac{\delta S}{\delta A_\nu} = \sum_\sigma\left[D_\mu\left(e^{-T^2/a}F^{\mu\nu}\right) + [T, e^{-T^2/a}D_\nu T]\right] = 0\,. \tag{4.3}$$

Here $\sum_\sigma$ denotes the symmetrization of U(2) matrices according to the above prescription of Str.

Now let us look for a solution of these equations. Due to the symmetrization, it is possible to find out a solution which is a precise analogue of the one of branes ending on a brane previously discussed:

$$T = x_9\mathbb{1} + \Phi\,, \qquad A_i = \epsilon_{aij}x_j\frac{W(r)}{r}\frac{1}{2}\sigma_{a-5}\,, \tag{4.4}$$

where a, i and j run from 6 to 8, and σ_{a-5}'s are sigma matrices. The adjoint scalar field Φ is given by

$$\Phi = x_a\frac{F(r)}{r}\frac{1}{2}\sigma_{a-5}\,. \tag{4.5}$$

The scalar field Φ and the gauge fields A_i in fact take the form of the Prasad-Sommerfield limit of the t'Hooft-Polyakov monopole solution,

$$F(r) = \frac{C}{\tanh(Cr)} - \frac{1}{r}\,, \qquad W(r) = \frac{1}{r} - \frac{C}{\sinh(Cr)}\,. \tag{4.6}$$

Note that accordingly the solution satisfies the original Bogomol'nyi equation $B_i + D_i\Phi = 0$.

4.2 The energy from the Bogomol'nyi bound

Due to the symmetrized trace prescription, the Bogomol'nyi bound argument goes in almost the same manner as the Abelian case in the previous section. The energy is reorganized as

$$\mathcal{E} = \mathcal{T}\,\mathrm{Str}\int d^9x e^{-T^2/a}\left(\mathbb{1} + (\partial_9 T)(\partial_9 T) + (D_iT)(D_iT) + B_iB_i\right)$$
$$= \mathcal{T}\,\mathrm{Str}\int d^9x e^{-T^2/a}\left((\mathbb{1}-\partial_9T)^2 + 2\partial_9T + (D_iT+B_i)^2 - 2(D_iTB_i)\right). \quad (4.7)$$

Strictly speaking, this is not really the bound, for the quantities $(\mathbb{1}-\partial_9T)^2$ and $(D_iT+B_i)^2$ are not quite the perfect square. The exponential factor of the tachyon could possibly be appearing inside the perfect square, due to the symmetrized trace prescription. In another word, only if the two quantities

$$\mathbb{1}-\partial_9T, \quad D_iT+B_i \quad (4.8)$$

were commutating with the tachyon T, the above expression (4.7) would give the energy bound. Our solution actually satisfies the BPS equations, which means nothing but the vanishing of the above two quantities (4.8). Thus eq. (4.7) indeed provides the bound of the energy.

Let us evaluate the energy from this expression (4.7). For our solution, the energy is given by the topological quantities and we have

$$\mathcal{E} = \mathcal{T}\,\mathrm{Str}\Bigg(2\sqrt{\pi a}\int d^8x\ \mathbb{1}\left[S(T/\sqrt{a})\right]^{\infty}_{x_9=-\infty} -$$
$$- 2\sqrt{\pi a}\int d^5x\int dx_9\int_{r=\infty} dS_i\left[S(T/\sqrt{a})B_i\right]\Bigg), \quad (4.9)$$

where, for this expression to be well-defined, we defined the error function by its Taylor expansion around the origin. The first term of eq. (4.9) gives the energy of two parallel D8-branes, due to the trace of the unit matrix $\mathbb{1}$. Here the tachyon field is dominated by $T\sim x_9\mathbb{1}$, thus the step function of the matrix argument T reduces to the ordinary Abelian step function.

The evaluation of the second term of eq. (4.9) is non-trivial, as it should correspond to the energy of a D6-brane which is not simply spread over the entire space but suspended between two parallel D8-branes along the x_9 axis, as we will see below. To compute this term, let us diagonalize the scalar field Φ by using the gauge symmetry so that

$$T = \begin{pmatrix} x_9 + C/2 & 0 \\ 0 & x_9 - C/2 \end{pmatrix}, \quad (4.10)$$

where we have already expanded the function $F(r)$ around $r\sim\infty$ and neglect the higher order terms which is dumping exponentially in r. This diagonal form (4.10) of the tachyon indicates that two parallel D8-branes are located at $x_9=\pm C/2$. At this gauge the magnetic field at $r\sim\infty$ also becomes diagonal:

$$B_i \sim -\frac{1}{2}\frac{x_i}{r^3}\sigma_3\,. \quad (4.11)$$

Therefore we can evaluate the energy explicitly as

$$\begin{aligned}\mathcal{E}_{\rm D6} &= 2\sqrt{\pi a}V_{12345}\mathcal{T}\mathrm{Str}\left[\int dx_9 \begin{pmatrix} S\left((x_9+C/2)/\sqrt{a}\right) & 0 \\ 0 & S\left((x_9-C/2)/\sqrt{a}\right)\end{pmatrix} 2\pi\sigma_3\right] \\ &= 4\pi\sqrt{\pi a}V_{12345}\mathcal{T}\int dx_9 \left(S\left((x_9+C/2)/\sqrt{a}\right) - S\left((x_9-C/2)/\sqrt{a}\right)\right). \end{aligned} \tag{4.12}$$

The step function properly accounts that the energy density of a D6-brane is localized only on the line segment $-C/2 < x_9 < C/2$. Thus this is consistent with the proposed D-brane configuration.

The RR-charge of this solution can be evaluated by using the formula (2.7) for the CS term. One ends up with an appropriate distribution of the RR-charges. In particular we can manifestly see the D6-brane charge localized on the line segment on the x_9 axis. As discussed in section 3.3, the CS term in this case again turns out to be the same as the BPS bound energy, though up to the ordering ambiguity.

4.3 Non-commutative monopole

The non-commutativity on the worldvolume of D-branes are equivalent to turning on the background NS-NS b-field [41]. This constant b-field can be thought of as a constant magnetic field on the D-brane, so this leads us to the brane bound states in section 2. Therefore, in terms of the brane configuration, the non-commutative monopole can be interpreted as a D6-brane ending on a D8-D6 bound state [42]. Since we have already studied the D6-brane ending on a D8-brane as well as a D8-D6 bound state, as in (3.1) and (3.2) and in (2.9) respectively, it is very easy to combine these solutions together. For the U(1) non-commutative monopole [43, 44, 45], the corresponding solution is

$$T = x_9 - \frac{s}{r} + b_{78}x_6, \quad B_i = \frac{sx_i}{r^3} + b_{78}\delta_{i6}\,, \tag{4.13}$$

where b is the background b-field. All the analysis of the energy and the Bogomol'nyi bound are going well, reproducing the expected results of the energy of the non-commutative monopole. Also the generalization to the U(2) non-commutative monopole [42, 44, 46, 47] is trivial.

5. Conclusions, discussions and future directions

Let us summarize our results. We obtained various brane configurations as classical solutions of the tachyon model (1.1) of Minahan and Zwiebach, which include (i) a D8-D6 bound state, (ii) a (F, D8) bound state, (iii) a string junction, (iv) D6-branes ending on a D8-brane, (v) a D6-brane suspended between two parallel D8-branes and (vi) their non-commutative generalizations. We computed the energies and the RR-charges of our solutions, and found that they showed appropriate distributions in space which was expected from the corresponding D-brane pictures. Also we found that the solutions (iv), (v) and (vi) satisfy the BPS equations, saturating the energy bounds. There is a trivial generalization of our results (i)–(vi) to the lower dimensional case, though by starting with the lower dimensional analogue of the tachyon model (1.1).

We have not checked whether the energies and the RR-charges of our solutions precisely match the ones in string theories, though we have found their distributions to be in nice agreements with the expected results. However it is anticipated that the energies and the RR-charges will have the correct values only when we include the higher-derivative corrections to the action (1.1). We expect that the qualitative forms of our solutions will not be modified by the inclusion of the higher derivative corrections, as our solutions are supposed to preserve some fractions of supersymmetries in the spacetime.[7] In our forthcoming paper [27], we will study the higher derivative effects by making use of the BSFT action explicitly. We will find the perfect agreement of the energies and the RR-charges with the expected string theory results. Furthermore, we are going to find more variety of solutions such as a Dp-D($p-4$) bound state [49], (F, D($p-2$)) bound states ending on Dp-brane and a dielectric branes [50, 51].

There are many other interesting brane configurations, other than those considered in this paper. We will make brief comments on some of them.

- Nahm construction of a D8-brane from multiple D6-branes [39].

  To obtain a D6-brane in the tachyon model, we have to prepare at least two D9-branes. The D6-brane solution is given by the ABS construction [11] as $T = q\sigma_{i-5}x_i$ ($i = 6, 7, 8$). As in [28], we know that the D8-brane on which D6-branes are ending can be realized as a solution of the Nahm equation (which was shown to be a BPS equation on the effective theory of D6-branes)

  $$\partial_9 \Phi_i = \epsilon_{ijk}\Phi_j\Phi_k \,, \tag{5.1}$$

  where Φ_i are the adjoint scalars on the D6-branes. A solution for the SU(2) case is given by $\Phi_i = -\sigma_i/x_9$. We need at least two D6-branes to have a non-trivial solution of the Nahm equation (5.1), as is obvious. It in turn means that we must have at least four unstable D9-branes. It is conceivable to think of the adjoint scalar Φ_i as the deformation of the tachyon profile, as we have done in our construction of branes ending on branes. Now we have a conjecture for the tachyon profile as

  $$T = q(\sigma_i x_i \otimes \mathbb{1} - \sigma_i \otimes \Phi_i) = q(\sigma_i x_i \otimes \mathbb{1} + \sigma_i \otimes \sigma_i/x_9) \,, \tag{5.2}$$

  where the gauge field will not be necessary.

[7] It might be interesting to investigate how we can see supersymmetries explicitly in the tachyonic field theory, as analyzed in [48].

- Two D6-branes intersecting with each other, sharing 4+1 dimensional world-volume.

  This configuration requires at least two D9-branes. Our conjecture for the solution is

$$T = q\left[\sigma_1\left(x_6 - s\frac{x_4}{x_4^2+x_5^2}\right) + \sigma_2\left(x_7 + s\frac{x_5}{x_4^2+x_5^2}\right) + \sigma_3 x_8\right]. \tag{5.3}$$

  Two D6-branes are oriented along the 45 and 67 planes respectively, while sharing the spacetime along 01239. Our conjecture is based on the following argument: Let us first prepare a single D6-brane as $T = q\sigma_{i-5}x_i$. Then the fluctuation about this tachyon profile can be taken into account as $T = q\sigma_{i-5}(x_i - \Phi_i)$. The other D6-brane will be realized by turning on the adjoint scalars with

$$\Phi_6 + i\Phi_7 = \frac{s}{x_4 + ix_5}\,. \tag{5.4}$$

  This is consistent with the supersymmetric (=holomorphic) membrane embedding in the spacetime,

$$zw = s\,, \tag{5.5}$$

  where $z \equiv x_6 + ix_7$ and $w \equiv x_4 + ix_5$.

The final remark is concerning the fundamental strings in the closed string vacuum, which was discussed in references [52]–[55]. In section 2.2, we studied the string junction in which the fundamental string should have been appearing as a component of the three-pronged string, though we could not find the confined flux in our solution. In addition we have to note that it seems impossible to construct the fundamental strings ending on a D8-brane, originally studied in [35], in our tachyon model, though one might think we could do so similarly to the construction of D6-branes ending on a D8-brane in section 3. This might be related to the fact that the fundamental strings cannot be seen easily in our model. We will be faced with the same difficulty also in the case of the (F, D6) bound state ending on a D8-brane or suspended between two parallel D8-branes. These are simply dyonic BIons [36], but it seems hard to construct these configurations in our tachyon model.

We leave these issues for the future work.

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